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Compound Choking of a Two-Parallel Stream Through a Convergent Duct

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The choking of dual subsonic streams flowing through a convergent duct in contact has been investigated experimentally and theoretically. The experiment was conducted by using blow-down wind tunnel. The condition, when the dual stream flow choking (compound choking) occurs at the nozzle exit, was explained by one-dimensional analysis of compound sound wave propagation. The experimental results for the condition of compound choking were compared with the prediction from theoretical analysis, and the schlieren optical method using the spark light source has been used to visualize the flowfield.

Key Words : Compound Choking, Compound Sound Wave, Dual Subsonic Streams, Schlieren Method

Nomenclature

A_i : Stream cross sectional area, $i=1, 2$
a : Speed of sound
c_i : Propagation velocity of compound sound wave, $i=1, 2$
P_e : Static pressure of duct exit
P_{0i} : Plenum chamber pressure, $i=1, 2$
α : Absolute propagation velocity of compound sound wave
γ_i : Ratio of specific heats, $i=1, 2$
ϕ : Plenum chamber pressure ratio $(=p_{02}/p_{01})$

Subscripts

1 : High speed subsonic stream
2 : Low speed subsonic stream

1. Introduction

A one-dimensional analysis for the choking phenomenon of a single stream compressible flow has been already recognized and established. When the choking of a single stream flow occurs in a convergent nozzle, the stream velocity at nozzle exit is sonic and the mass flow rate depends only on the upstream stagnation pressure. An analogous phenomenon can occur in a single convergent nozzle contains dual streams flowing side by side. This choking phenomenon that we will call "compound choking (Bernstein et al., 1967)" of the dual streams flowing through a singe nozzle has been investigated over a long period of time (Bernstein et al., 1967 ; Hoge and Segars, 1965 ; Pearson et al., 1958 ; Fage, 1976), and partly inspired by the effort to develop the scramjet engine technology (Lewis and Hastings, 1987 ; Schindel, 1999) and supersonic ejector pump (German et al., 1966 ; Clark, 1995) and so forth. According to the previous works, it was pointed out that the respective stream Mach numbers at nozzle throat are not equal to one; one flow is supersonic and the other is subsonic, when the compound choking occurs. However, there are few particularly general mechanisms to explain the characteristics of compound choking, and currently available experimental evidence for

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compound choking is not definitive. In addition, most reports have dealt with the dual stream flow composed of supersonic and subsonic streams without appreciable mixing. The present study therefore aims at presenting the experimental and theoretical compound choking conditions of the subsonic and subsonic dual stream flows with considerable mixing through a long convergent duct. The theoretical compound choking condition was analyzed by examining the concept of compound sound wave propagation. For the experimental investigation, the side-view schlieren photographs were taken as well as the measurements of a pressure. Finally, the experiment results were compared with the theoretical predictions.

2. Theoretical Compound Choking Condition

Since a stream flowing in the nozzle is choked, when the pressure communication between the upstream and downstream flows is interrupted by sonic flow at the throat, the compound choking condition also can be explained by examining the small pressure disturbance propagates in the nozzle which contains the dual stream flow. As shown in Fig. 1, it is considered that the small pressure disturbance propagates through still two-layer gases forward the right side in the quasi-one-dimensional passage. In this case each stream velocities u_1, u_2 are zero and it will to be assumed that gases in contact have same static pressure p and different specific heat ratios γ and densitiesρ. This small pressure disturbance that we will call "compound sound wave" propagates with absolute velocity $\alpha \equiv c$ and downstream static pressure of this wave is also assumed to be same for whole cross section. Based on these assumptions, applying the continuity equation, the momentum equation, isentropic relation to a coordinate system attached to compound sound wave, the velocity of compound sound wave propagates through still gases can be obtained as

$$c=\sqrt{\frac{\gamma_2 A_1 a_1^2+\gamma_1 A_2 a_2^2}{\gamma_2 A_1+\gamma_1 A_2}} \tag{1}$$

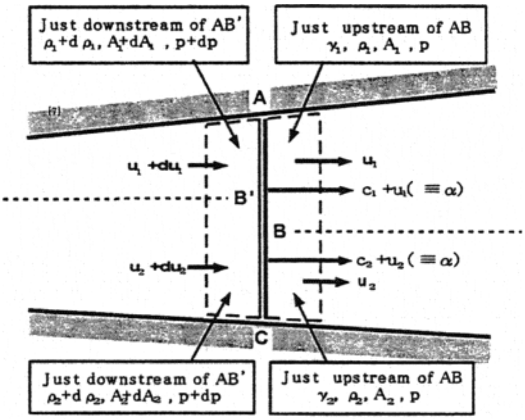


Fig. 1 Compound sound wave propagation

Furthermore, in order to determine the condition of the compound choking, the compound sound wave propagates through the dual streams flowing with respective velocities u_1 and u_2 is considered. As shown in Fig. 1, it is assumed that compound sound wave propagates with an absolute velocity $\alpha \equiv c_1+u_1=c_2+u_2$, although a respective propagation velocities are different as c_1, c_2. The assumptions for the conditions of pressure, specific heat ratio and density are same as the above still gases case. The continuity equation, the momentum equation and isentropic relation were also applied to a coordinate system attached to compound sound wave, and the velocity of compound sound wave in the quasi-one-dimensional passage is shown as

$$\frac{A_1}{\gamma_1}\left\{\left(\frac{a_1}{c_1}\right)^2-1\right\}+\frac{A_2}{\gamma_2}\left\{\left(\frac{a_2}{c_2}\right)^2-1\right\}=0 \tag{2}$$

We recognize that the compound choking occurs, when the absolute velocity of the compound sound wave α becomes zero, that is, the respective velocities of the compound sound wave c_1, c_2 are equal to the respective stream velocities u_1, u_2. Above equation makes it possible to confirm that when the dual stream flow is choked, one flow is supersonic and the other is subsonic.

3. Experimental Setup and Procedures

The experiments were performed in the

Gasdynamics Laboratory at Kyushu University at Japan. The schematic diagram of the pressure measuring and the optical system used in present work is shown in Fig. 2. The experiment was conducted by using the blow-down wind tunnel and working gas was air. A respective air in reservoirs of 30ata and 10ata flows into the plenum chamber separated by the splitter plate. Each air is stagnated in each plenum chamber and enters in the single convergent duct in contact. This dual stream flow composed with the upper high velocity stream and the lower low velocity stream is discharged into the atmosphere through the settling chamber. As shown in Fig. 2, pressures at each plenum chamber and the duct exit were measured to determine the compound choking condition. The configuration and dimensions of experimental apparatus are shown in Fig. 3. The cross sections of the convergent duct and the straight duct are rectangular for the side-view optical investigation. The widths of the plenum chamber, the convergent nozzle and the straight duct are constant at 80mm. The heights of convergent duct exit and inlets separated by the splitter plate are 20mm in all, and the length of the convergent duct is 155mm. As shown in Fig. 2 the spark schlieren optical method was used to visualize the flow field and the duration time of the Xe-lamp is 20 ns. Since we considered as the condition of the compound choking that, the pressure at the convergent duct exit begins to increase in proportion to the increase of the plenum chamber pressure, and the ratio of the plenum chamber pressure to the duct exit pressure begins to be maintained constant, the experiments were accomplished by keeping the ratio of the upper plenum chamber pressure p_{01} to the lower plenum chamber pressure p_{02} constant and by increasing each plenum chamber pressure gradually. In addition to the pressure measurements, the flowfields were visualized for given pressure ratios. In the present work, the stagnation pressure ratio $\phi = p_{02}/p_{01}$ was varied from about 1.0 to 0.7 and the pressure of settling chamber was nearly equal to the atmospheric pressure during the experiment.

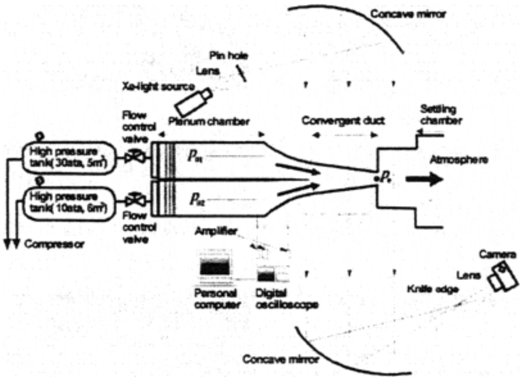


Fig. 2 Experimental apparatus

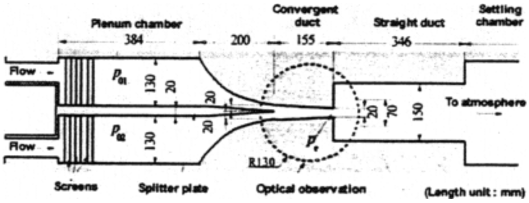


Fig. 3 Configuration of test section

4. Results and Discussion

4.1 Flow visualization

As described above, the visualizations were carried out by keeping the stagnation pressure ratio ϕ constant and by increasing p_{01} and p_{02} gradually. Figure 4 shows schlieren photographs for the optical observation region of the convergent duct shown in Fig. 3 for $\phi = 0.9$. In addition, the different directions of the knife-edge were adjusted so that the density gradients in different directions were examined.

Figure 4 a-v and b-v taken with vertical knife edge for the pressure ratio $p_{01}/p_e = 1.84$ and 1.92, respectively, show that the dual stream flow is not choked, and the streams from the duct exit are subsonic dual stream flows, since there are no shocks in the jet from the duct exit. Fig. 4 c-v and d-v were taken for the plenum chamber pressures $p_{01} = 200$kPa and 210kPa, respectively, and for pressure ratio p_{01}/p_e 1.96 in all. These show shock waves in the downstream from the duct exit. In these cases, the duct exit pressure is greater than the atmospheric pressure and p_{01}/p_e is maintained nearly constant for the increase of p_{01}. Conse-

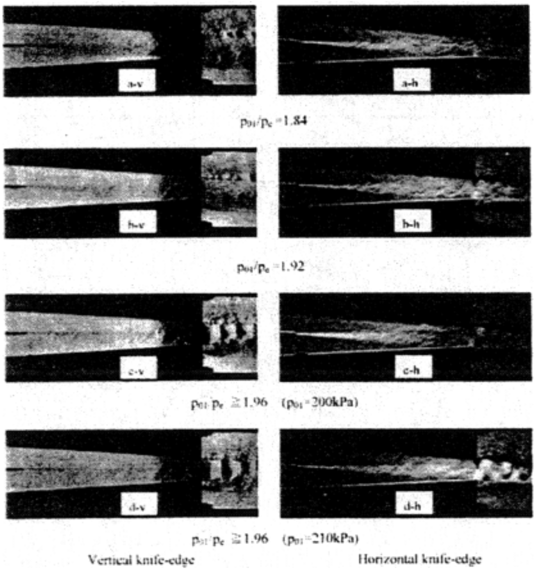
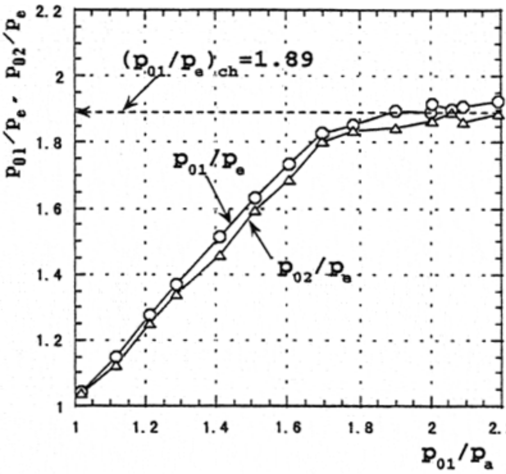


Fig. 4 Schlieren photographs, $\phi(\equiv p_{02}/p_{01})=0.9$

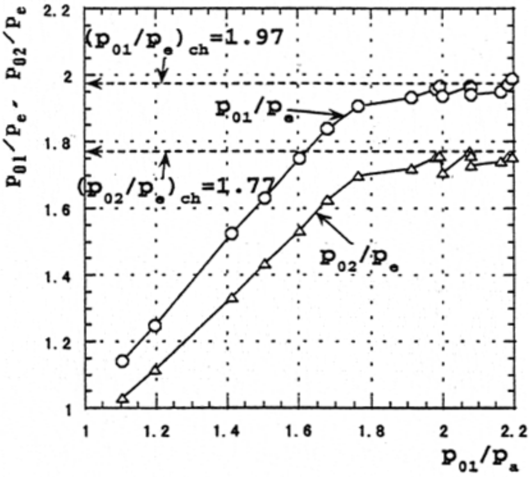
quently these dual stream flows in the duct are under the under expansion condition, and these flow are considered to choke at the convergent duct exit. Figure 4 c-h and d-h with horizontal knife-edge show the considerable mixing region between two streams. This mixing region induced by different velocity streams suggests the disagreement between the theoretical and experimental choking conditions. According to above examines of the schlieren photographs, the pressure ratio of the compound choking condition is considered to exit between $p_{01}/p_e=1.96$ and 1.92. The more detailed choking condition placed in this extent was confirmed with the measurement of the plenum chamber pressures and the duct exit pressure. The measured pressure ratios indicating the compound choking condition for various ϕ are shown in following chapter.

4.2 Pressure measurements

Figure 5 shows the relationship between stagnation pressures at plenum chambers upstream of the convergent duct and static pressure at the duct exit. The abscissa is the ratio of stagnation pressure p_{01} to atmosphere p_a, and the ordinate is the ratio of stagnation pressure at plenum chamber p_{01} or p_{02} to static pressure at the duct exit p_e. It will be seen that as p_{01}/p_a increases there is a rapid rise in p_{01}/p_e and p_{02}/p_e, but these ratios rapidly reach a fixed value after which they do not change. Along the whole of the flat portion of this characteristics the flow pattern in the duct is fixed and the system can be regarded as choked.



(a) $\phi=1.0$



(b) $\phi=0.9$

Fig. 5 Relationship between plenum pressure and duct exit static pressure

4.3 Compound choking condition

Figure 6 shows the experimental and one-dimensionally calculated compound choking conditions with $(p_{01}/p_e$-$p_{02}/p_e)$ plane for the various stagnation pressure ratio ϕ. The solid line

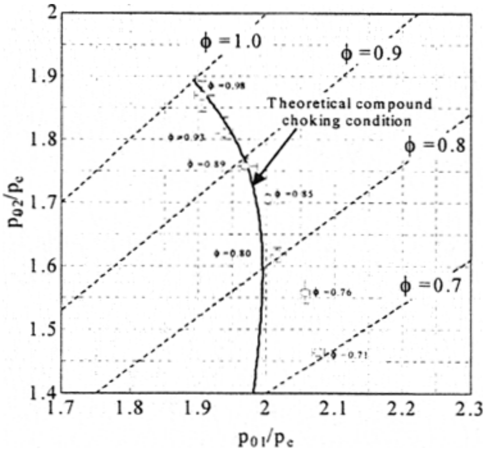


Fig. 6 Comparison of experimental and theoretical compound choking conditions ($A_e/A_{In}=A_{1In}/A_{In}=0.5$, $T_{02}/T_{01}=1.0$, Air/Air)


ϕ =0.9
ϕ =0.7

Fig. 7 Schlieren photographs, $p_{01}=190$kPa

shows the calculated choking condition based on the assumptions of two one-dimensional, isentropic, frictionless, and unmixed subsonic air streams flowing in contact through the convergent duct. The static pressure and total temperature at the whole cross section of the duct were assumed constant. Based on above assumptions, the continuity equation and the isentropic relation were applied to the same duct shape used in experiments. In this calculation, compound choking was assumed to occur at the pressure ratio at which the absolute velocity of compound sound wave becomes zero, so the calculated Mach numbers and stream areas of the dual stream at given duct exit satisfy Eq. (2). The pressure measurements at plenum chambers and duct exit were carried out for the cases of $\phi=0.98$, 0.93, 0.89, 0.85, 0.80, 0.76 and 0.71. When the compound choking occurs, the duct exit pressure increases in proportion to the increase of respective chamber pressures, so pressure ratios, p_{01}/p_e and p_{02}/p_e are maintained constant. Circular symbols shown in Fig. 6 present these experimental compound choking conditions for each stagnation pressure ratio ϕ. The comparison of the experimental results and one-dimensional isentropic calculations shows a good agreement until ϕ becomes almost 0.9. On the other hand, in the case of $\phi<0.9$, considerable disagreements occur and this tendency was on the enlargement in proportion to the decrease of ϕ. These disagreements are considered to be due to the mixing effect depends on the difference of each stream mass flow and the duct length and so on. By our optical observation, the mixing regions between two streams were also confirmed to be thickened in proportion to the decrease of ϕ, that is, the increase of the difference of each stream mass flow. These tendencies clarified through the optical observation are shown in Fig. 7 for the case of $\phi=0.9$, 0.7 and the higher chamber pressure $p_{01}=190$kPa in all. The schlieren photographs were taken with a horizontal knife edge and for the compound choking state.

We did not measure the Mach number distribution at the duct exit, but it was possible to know roughly that in spite of the existence of the mixing region, the high speed stream is supersonic at the duct exit because of the greater valve of p_{01}/p_e than 1.893 and the low speed stream is subsonic because of the lower value of p_{02}/p_e than 1.893. This means the high speed subsonic stream was choked in the convergent duct independently between the duct wall and the low speed subsonic stream. But a complete analysis as to the compound choking condition requires the information for the rate of growth of the mixing region between the dual streams, frictional effect and three-dimensional effect and so on.

5. Conclusion

The choking phenomenon of dual subsonic streams flowing in contact through a single convergent duct was investigated experimentally and theoretically. The results are summarized as follows.

(1) According to the analysis of the compound sound wave propagation and the experiment of

pressure measurement when the compound choking phenomenon occurs, the respective stream Mach numbers are not equal to one; one stream is supersonic and the other stream is subsonic.

(2) The experimental and theoretical compound choking conditions for dual subsonic streams agree with each other while the stagnation pressure ratioϕ is greater than about 0.9, on the other hand, in the case$\phi<0.9$ the considerable disagreements occur due to the mixing effect.

(3) The mixing effect is enlarged in proportion to the decrease of the stagnation pressure ratioϕ.

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